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Wave drag of rapidly and horizontally moving Rankine ovoid in uniformly stratified fluid

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Abstract

A theory on the wave drag as the Rankine ovoids moving horizontally, uniformly and rapidly in uniformly vertical stratified fluid (or ocean) is presented. A mass source resulting from the theory in a uniform fluid is used to model a hydrodynamic interaction between the Rankine ovoid and stratification. Theoretical results show that there exists an asymptotic state of the drag in supercritical regimes where internal Froude numbers are large. When the Rankine ovoid reduces to a sphere, the result we obtained is in good agreement with the previous theoretical and experimental results. An experiment on the elongated Rankine ovoid is also carried out to validate the theoretical results.

Keywords: Internal wave; Stratified flow; Drag; Rankine ovoid; Internal Froude number; Source distribution

1. Introduction

Unlike the case in the uniform fluids, the internal gravity waves in the vertically stratified fluid (or ocean) can propagate not only at sea surface but also in fluids (or ocean). For internal waves with a small amplitude generated by a uniformly moving body, the body can be replaced by an equivalent distribution of mass source, such as combination of source and sink and dipole. Thus, for a given point mass source, solution of the linear problem for this kind of the internal wave's generation can be found by superposition of an elementary special solution-retarded Green's function [1,2]. Previously, Gorodtsov and Teodorovich [1] and Greenslade [3], respectively, carried out the theoretical study of drag of sphere moving horizontally and uniformly in the fluid with the constant Brunt-Väisälä frequency. Lofquist and Purtell [4] and Xu et al. [5], respectively, reported the experimental studies of the drag increments for the sphere and the Rankine ovoid moving horizontally and uniformly in stratified fluids.

In this paper, we present the intrinsic simplification of the problems of internal waves generated by horizontally, rapidly, and uniformly moving Rankine ovoid in the uniformly stratified fluid and the asymptotic solutions in detail. The basic equations are formulated in Section 2, whereas the general theory of the drag increment in the case of the uniformly stratified fluid is given in Section 3. The theory of the drag increment of Rankine ovoid moving uniformly and rapidly in uniformly stratified fluid is given in Section 4. A reduced case as the Rankine ovoid becomes a sphere is discussed in Section 5, and some comparisons and conclusions are presented in Section 6.

2. Basic equations

Assuming $\bar{\rho}$ is the mean density and is a constant, $\rho_0(z)$ the vertical density profile, ρ a perturbation of the

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environmental density $\rho_0(z)$, $\boldsymbol{q}$ velocity of fluid, and p pressure. The equations of a linearized small perturbation describing the conservation of mass and momentum and incompressibility are

$$\begin{aligned}&\rho_t + \bar{\rho}(N^2/g^2)(\boldsymbol{q}\cdot\boldsymbol{g}) = 0,\\ &\bar{\rho}\boldsymbol{q}_t = -\nabla p + \rho\boldsymbol{g}, \quad \nabla\cdot\boldsymbol{q} = 0.\end{aligned} \tag{1}$$

Here, the change of the density is ignored in the inertial terms, i.e. the so-called Boussinesq's approximation is used. Let z be positive along with the gravity, the Brunt-Väsälä's frequency is $N^2 = (g/\bar{\rho})\mathrm{d}\rho_0/\mathrm{d}z$. For the mass source $m(\boldsymbol{r},t)$, in which $\boldsymbol{r} = (x,y,z)$, a quasi-perturbation potential ϕ can be introduced as follows:

$$\mathcal{L}\phi(\boldsymbol{r},t) = m(\boldsymbol{r},t), \quad \mathcal{L} = \partial_t^2\Delta + N^2\Delta_h, \tag{2}$$

where Δ and Δ_h are the Laplace and horizontal Laplace operator, respectively. Hence, the internal perturbations $\boldsymbol{q}$, p and ρ can be represented as

$$\boldsymbol{q} = \boldsymbol{l}_1\phi, \quad p = l_2\phi, \quad \rho = l_3\phi, \tag{3}$$

where

$$\begin{aligned}&\boldsymbol{l}_1 = \partial_t^2\nabla + N^2\nabla_h, \quad l_2 = -\bar{\rho}(\partial_t^2 + N^2),\\ &l_3 = -\bar{\rho}(N^2/g)\partial_{tz}^2\end{aligned} \tag{4}$$

in which ∇ and ∇_h are Hamiltonian and horizontal Hamiltonian operator, respectively. By the Green function $G(\boldsymbol{r},t)$, an inhomogeneous equation describing the internal waves with Dirac δ in r.h.s is derived

$$\mathcal{L}G(\boldsymbol{r},t) = \delta(\boldsymbol{r})\delta(t) \tag{5}$$

and the quasi-perturbation potential ϕ can be represented by a convolution of the Green function G with the mass source $m(\boldsymbol{r},t)$. Similarly, the perturbation pressure can be denoted by

$$p(\boldsymbol{r},t) = -\bar{\rho}(\partial_{tt}^2 + N^2)\partial_t\int G(\boldsymbol{r}-\boldsymbol{r}',t-t')m(\boldsymbol{r}',t')\mathrm{d}\boldsymbol{r}'\mathrm{d}t', \tag{6}$$

where the Green function G in Eq. (6) is the so-called retarded Green function, and it is converted into zero in negative time, i.e. the Green function satisfies the causality condition [1]. The causality avoids an additional selection of radiant condition.

For the internal perturbations, a significant result of Eq. (3) is an energy balance including the kinematic and potential energy E, energy flux $\boldsymbol{S}$ and work from the mass source W, namely,

$$\begin{aligned}&\partial_t\int E\mathrm{d}\boldsymbol{r} + \int \boldsymbol{S}\mathrm{d}\boldsymbol{s} = \int p(\boldsymbol{r},t)m(\boldsymbol{r},t)\mathrm{d}\boldsymbol{r} \equiv W,\\ &E = \bar{\rho}|\boldsymbol{q}|^2/2 + (g\rho)^2/(2\bar{\rho}N^2), \quad \boldsymbol{S} = p\boldsymbol{q}.\end{aligned} \tag{7}$$

For the mass source, i.e. the equivalent body (for instance, the Rankine ovoid) moving at a constant velocity, the perturbation energy in full space is unchanged with time, so the wave generation is stationary. Hence, for an estimation of radiant energy of the internal waves, one of the integrals in Eq. (7), i.e. p – integral in all the wave properties is enough. The energy loss in a unit time W can be calculated simply as the mass source is prescribed *in priori*, therefore a significant task is to find the pressure p and to calculate the integral in full space.

3. Drag of rapidly moving mass sources in stratified fluid

In order to solve the drag of the internal waves generated by the Rankine ovoid, the Fourier transformation is used in this study. Let $f = f(\boldsymbol{r},t)$ be an arbitrary physical quantity in this study, thus the Fourier transform pairs with respect to frequency ω and wave number $\boldsymbol{k}$ are defined as follows:

$$\begin{aligned}f(\boldsymbol{r},t) &= \frac{1}{2\pi}\int f_\omega(\boldsymbol{r})\exp(-\mathrm{i}\omega t)\mathrm{d}\omega = \frac{1}{(2\pi)^n}\int f_k(t)\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})\mathrm{d}\boldsymbol{k}\\ &= \frac{1}{(2\pi)^{n+1}}\int f_{\boldsymbol{k},\omega}\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r} - \mathrm{i}\omega t)\mathrm{d}\boldsymbol{k}\mathrm{d}\omega\end{aligned} \tag{8}$$

and correspondingly

$$\begin{aligned}&f_\omega(\boldsymbol{r}) = \int f(\boldsymbol{r},t)\exp(\mathrm{i}\omega t)\mathrm{d}t, \quad f_{\boldsymbol{k}}(t) = \int f(\boldsymbol{r},t)\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})\mathrm{d}\boldsymbol{r}\\ &f_{\boldsymbol{k},\omega} = \int f(\boldsymbol{r},t)\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r} + \mathrm{i}\omega t)\mathrm{d}\boldsymbol{r}\mathrm{d}t\end{aligned} \tag{9}$$

It should be pointed out that all the integrals above are calculated from $-\infty$ to $+\infty$ except the frequency-Fourier transformation of the retarded Green function. The integral of this function with respect to the time t is shortened to a half infinity region $(0,+\infty)$, which leads to an analyticity of the retarded Green function in the upper half plane of the complex frequency.

For the mass source moving at a uniform velocity $\boldsymbol{v}_0$, it follows that

$$\begin{aligned}&m(\boldsymbol{r},t) = M(\boldsymbol{R}), \quad \boldsymbol{R} \equiv \boldsymbol{r} - \boldsymbol{v}_0 t\\ &m_{\boldsymbol{k}} = M_{\boldsymbol{k}}\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}_0 t), \quad m_{\boldsymbol{k},\omega} = 2\pi M_{\boldsymbol{k}}\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v}_0).\end{aligned} \tag{10}$$

Obviously, the full Fourier transformation $m_{\boldsymbol{k},\omega}$ is proportional to the Dirac function $\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v}_0)$, i.e. there exists a linear relation between the frequency ω and wave number $\boldsymbol{k}$. By the stationary character of internal wave fields in the body frame, the pressure is found,

$$\begin{aligned}p = &\frac{\mathrm{i}}{(2\pi)^n}\bar{\rho}\\ &\times\int \omega(N^2 - \omega^2)G_{\boldsymbol{k},\omega}M_{\boldsymbol{k}}\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{R})\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v}_0)\mathrm{d}\boldsymbol{k}\mathrm{d}\omega.\end{aligned} \tag{11}$$

Because the wave drag of the mass source moving at the uniform velocity is equal to the energy loss along with a unit path, the wave drag can be defined by

$$D = \frac{W}{v_0} = \frac{1}{v_0}\int p(\boldsymbol{r},t)m(\boldsymbol{r},t)\mathrm{d}\boldsymbol{r}. \tag{12}$$

Using the Fourier transform yields

$$D = \frac{\mathrm{i}}{(2\pi)^n}\frac{\bar{\rho}}{v_0}\int \omega(N^2-\omega^2)G_{\boldsymbol{k},\omega}|M_{\boldsymbol{k}}|^2\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v}_0)\mathrm{d}\boldsymbol{k}\mathrm{d}\omega, \tag{13}$$

where n is the dimension number of space. For the real retarded Green function, its Fourier transform as in Eqs. (11) and (13) possesses some symmetric characters, i.e. it is even symmetric for the real part and odd symmetric for the imaginary part for the displacements of $\boldsymbol{k}\to-\boldsymbol{k}$ and $\omega\to-\omega$. Based on this feature, a non-zero contribution of the wave drag D in the integral expression is given only by the imaginary part. For the uniformly stratified fluid, the imaginary part of the retarded Green function is concentrated on the dispersion curved surface of internal waves $\omega^2k^2=N^2k_h^2$, and is proportional to the corresponding Dirac delta function, namely,

$$\mathrm{Im}G_{k,\omega} = -\pi\,\mathrm{sgn}\omega\,\delta(\omega^2k^2-N^2k_h^2) \tag{14}$$

using Eq. (14) yields

$$D = \frac{\pi}{(2\pi)^n}\frac{\bar{\rho}}{v_0}\int |\omega|(N^2-\omega^2)|M_{\boldsymbol{k}}|^2\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v}_0) \times\delta(\omega^2k^2-N^2k_h^2)\mathrm{d}\boldsymbol{k}\mathrm{d}\omega. \tag{15}$$

From the above formulae, it is known that there exist two relations $\omega=\boldsymbol{k}\cdot\boldsymbol{v}$ and $\omega^2k^2=N^2k_h^2$ in this theory, so that the times of the integrals will reduce to two. In a three-dimensional space $n=3$, two residual integrals are only those along the wave vectors, thus it follows that

$$D = \frac{\pi}{(2\pi)^n}\frac{\bar{\rho}}{v_0}\int |\omega|(N^2-\omega^2)\sum_{i=1}^{4}|M_{\boldsymbol{k}i}|^2\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v}_0) \times\delta(\omega^2k^2-N^2k_h^2)\mathrm{d}\boldsymbol{k}\mathrm{d}\omega, \tag{16}$$

where

$$\boldsymbol{k}_1 = \left(\frac{\omega}{v_0},\frac{\omega}{Nv_0}\sqrt{k^2v_0^2-N^2},\frac{k}{N}\sqrt{N^2-\omega^2}\right). \tag{17}$$

For the $\boldsymbol{k}_i$ as $i=2,3,4$, unlike $\boldsymbol{k}_1$, only one sign is changed in the components. From the above discussion, it is known that an allowable frequency for the internal gravity waves generated by the mass sources is bounded above by Brunt-Väisälä frequency, whereas the allowable wave number is also bounded below by a minimum of wave number N/v_0.

4. Drag of horizontally and rapidly moving Rankine ovoids

A combination of the source and sink is considered in this section, which is a body model as the combination moves at a uniform velocity in the horizontal direction. For the uniform fluid, the bodies are called the Rankine ovoids. For horizontally moving Rankine ovoids, the source can be represented by [1]

$$m(X,\chi) = M(\boldsymbol{R}) = m_0\delta(\chi)[\delta(X-a)-\delta(X+a)], \tag{18}$$

where $X=x-v_0t, \boldsymbol{R}=(X,\chi)$, in which v_0 is the moving velocity of the source along with horizontal coordinate X, and $\chi=(y^2+z^2)^{1/2}$ is the radius in the cylinder coordinate; a is the distance from the origin of coordinates to the location of source and sink; m_0 is the intensity of source or sink. The Fourier transformation of the source (18) is

$$M_{\boldsymbol{k}} = -2\mathrm{i}m_0\sin(k_xa), \tag{19}$$

where k_x is the wave number along with x coordinate. From (18), it is deduced that the field of the internal waves generated by the Rankine ovoid is stationary.

In accord with the potential theory in uniform fluid, for weakly stratified fluid and uniformly moving source, the shape of the Rankine ovoid can be determined as follows:

$$\frac{\xi+1}{\sqrt{\eta^2+(\xi+1)^2}}-\frac{\xi-1}{\sqrt{\eta^2+(\xi-1)^2}} = \frac{2\pi a^2v_0}{m_0}\eta^2, \tag{20}$$

where $\xi\equiv X/a$ is a dimensionless horizontal coordinate (axial coordinate) and $\eta\equiv\chi/a$ is a dimensionless radius in the cylinder coordinates, by these definitions the maximal length of the Rankine ovoid is $2X_0$, and its maximal diameter is $2\chi_0$. To define a dimensionless quantity $\Pi\equiv m_0/(\pi a^2v_0)$, thus the surface shape of the Rankine ovoid will depend on the parameter Π and its maximal length, and the diameter can be determined by the following formulae:

$$\chi_0=\eta_0a,\quad X_0=\xi_0a,\quad \eta_0^2\sqrt{\eta_0^2+1}=\Pi. \tag{21}$$

In the following we give two limiting cases of the Rankine ovoid.

Case 1: As $2a\to0$ and the moment of the source–sink combination $d=2am_0(=\text{const})$ remains unchanged, the Rankine ovoid reduces to a sphere and the source–sink combination reduces to a dipole. Indeed, the dimensionless quantity Π increases infinitely as $2a\to0$, this leads to the determination of the radius of the sphere, i.e. $\chi_0=X_0=(d/2\pi v_0)^{1/3}$.

Case 2: As $2a\to\infty$ and the moment of the source–sink combination $d=2am_0(=\text{const})$ remains unchanged as in Case 1, the dimensionless quantity Π will become small. Similar to (19), the diameter of the Rankine ovoid becomes also small by rule $2\chi_0\approx(2d/\pi v_0a)^{1/2}\to0$ in the first order approximation, and its length increases by rule $2X_0\approx2a\to\infty$. An interesting case is that if the diameters of the Rankine ovoids remain unchanged, and if the intensity of source–sink combination also remains unchanged, i.e. $m_0=\text{const}$, the moment of the combination is assumed not to be a constant. Under this assumption, from (19), an invariable diameter $2\chi_0\approx2(m_0/\pi v_0)^{1/2}$ can be found, whereas its length increases with the increasing of the distance between the source and sink, i.e. $2X_0\approx2a$ asymptotically (the exact relation is $2X_0\approx2(a+\chi_0)$).

When the radius of the Rankine ovoid moving at a uniform velocity (χ_0) remains unchanged, only the distance between the source and sink $2a$ varies, a series of Rankine ovoids are formed. For these Rankine ovoids, two length scales, i.e. the vertical scale χ_0 and the horizontal scale a play an important part in the computation of the wave drag except the stratified scale N. To define an internal Froude number $F_a = v_0/(Na)$, one can represent the wave drag by means of this Froude number, namely,

$$D = \frac{\bar{\rho} m_0^2 N^2}{4\pi v_0^2} \Phi\left(\frac{2Na}{v_0}\right) \int_{N/v_0}^{\infty} \frac{\mathrm{d}k}{\sqrt{k^2 - N^2/v_0^2}},$$
$$\Phi(t) = 1 - \frac{2}{t} J_1(t), \tag{22}$$

where $J_1(t)$ is the Bessel function. Eq. (22) is an expression of the wave drag in the linear approximation. In this formula, the integral with respect to the wave number k increases logarithmically due to the large contribution from the short waves.

In order to eliminate the singularity in (22) at the infinite wave number, a regulation is carried out in this study, i.e. the above limit of the integral (16) is truncated at a finite wave number K, whereas the K is still large so that $K \gg N/v_0$. Thus, the theoretical wave drag (22) can be found as follows:

$$D = \frac{\pi}{4} \bar{\rho} \chi_0^4 N^2 \Phi\left(\frac{2}{F_a}\right) (\ln F_{\chi_0} + C), \quad F_{\chi_0} \gg 1, \tag{23}$$

where C is a constant, F_{χ_0} is another internal Froude number and defined by the radius χ_0, i.e. $F_{\chi_0} = v_0/(N\chi_0)$. And the $\Phi(\cdot)$ is a function related to the Bessel function. Comparing with the previous results, we inverse the wave drag to the dimensionless coefficient of the wave drag c_D, namely,

$$c_\mathrm{D} = \frac{D}{\frac{1}{2}\bar{\rho} v_0^2 S} = \Phi\left(\frac{2}{\varepsilon F_{\chi_0}}\right) \frac{\ln F_{\chi_0} + C}{2F_{\chi_0}^2}, \quad \varepsilon \equiv \frac{\chi_0}{a}, \tag{24}$$

where $S = \pi\chi_0^2$ is an area of the section perpendicular to the horizontally moving axis for the Rankine ovoids. ε is a ratio of the radius to length of the Rankine ovoids and is a small parameter for the elongated Rankine ovoids. Obviously, there exists a relation between two Froude numbers

$$F_a \equiv v_0/(Na) = \varepsilon F_{\chi_0}. \tag{25}$$

If one of the two Froude numbers is large, an asymptotic formula is obtained for the elongated Rankine ovoids,

$$D = \frac{\pi}{8} \bar{\rho} \chi_0^4 N^2 \frac{\ln F_{\chi_0} + C}{F_a^2}, \quad F_{\chi_0} \gg F_a \gg 1 \tag{26}$$

and the drag coefficient is

$$c_\mathrm{D} = \frac{D}{\frac{1}{2}\bar{\rho} v_0^2 S} = \frac{\ln F_{\chi_0} + C}{4F_a^2 F_{\chi_0}^2} = \left(\frac{a}{\chi_0}\right)^2 \frac{\ln F_{\chi_0} + C}{4F_{\chi_0}^4}. \tag{27}$$

From the asymptotic result (27), it can be expected that the value of drag coefficient is smaller when compared with non-regulation's result.

5. The reduced case: a sphere

As the elongation-to-radius ratio a/χ_0 of the Rankine ovoid becomes zero, the ovoid will reduce to a sphere, and its unique length scale is its radius R_0. Here, the theoretical results are reduced to those in Case 1. When $a \to 0$ and $d = 2am_0$ (the moment of dipole), and using the above regulation, one can find the wave drag of the sphere with radius R_0 as follows:

$$D = \frac{\pi}{8} \bar{\rho} R_0^4 N^2 \frac{\ln F_{R_0} + C}{F_{R_0}^2}, \tag{28}$$

where the internal Froude number of the sphere is defined as $F_{R_0} \equiv v_0/(NR_0) \gg 1$. Of course, (28) can be derived from the reduction of (26). However, because the wave drag of the Rankine ovoid includes two independent quantities χ_0 and a, whereas the sphere has only one independent quantity R_0, the reduction is not straightforward. By (28), the drag coefficient of the sphere can also be obtained, namely,

$$c_\mathrm{D} = \frac{\ln F_{R_0} + C}{4F_{R_0}^4}. \tag{29}$$

For the sphere, the constant C in (29) can be determined theoretically [1], that is

$$C = 7/4 - \gamma, \tag{30}$$

where $\gamma = 0.577$ is Euler's constant. Greenslade [3] constructed a composite theory to predict the drag increment in the regime of large Froude number. His theoretical results are in good agreement with the theoretical ones of Gorodtsov and Teodorovich [1] and with the experimental ones of Mason [6].

6. Determination of the constant C

An experiment is carried out in a Tank of Stratified Flow and Internal Wave with 6 m length, 0.5 m depth, and 1.8 m width, located at The Physical Oceanography Lab in OUC. In this experiment, the total depth of water is 40 cm, the densities on the surface and at the bottom are 1.00 and 1.03 g/cm^3, respectively, the vertical mean density is 1.015 g/cm^3. So the Brunt-Väisälä's frequency is 0.851 Hz. The internal Froude numbers F_r are taken from the following series: 0.95, 1.05, 1.33, 1.43, 1.45, 1.65, and 1.85 corresponding to the towed velocities 9.69, 10.71, 13.57, 14.59, 14.79, 16.83, and 18.77 (cm/s), respectively. The length-to-radius ratio is a fixed value 1.5. By these parameters, the experiment of the drag is performed in the linearly stratified flow. The theoretical and experimental results are compared, and are indicated in Figs. 1 and 2 gives a theoretical prediction of the drag coefficients

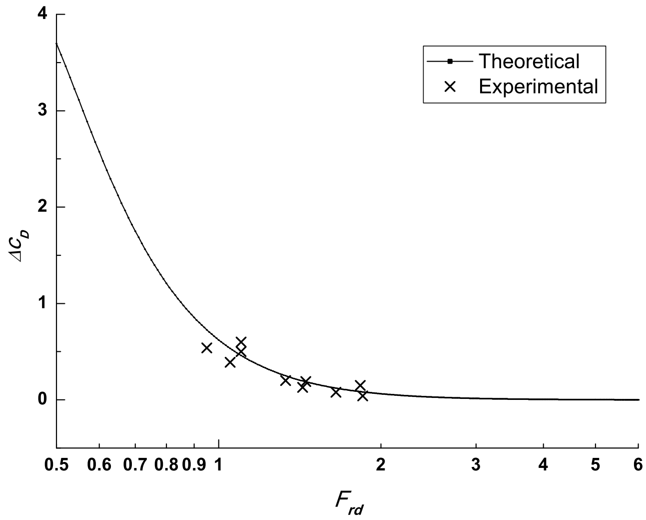

Fig. 1. A comparison between the theoretical and experimental results.

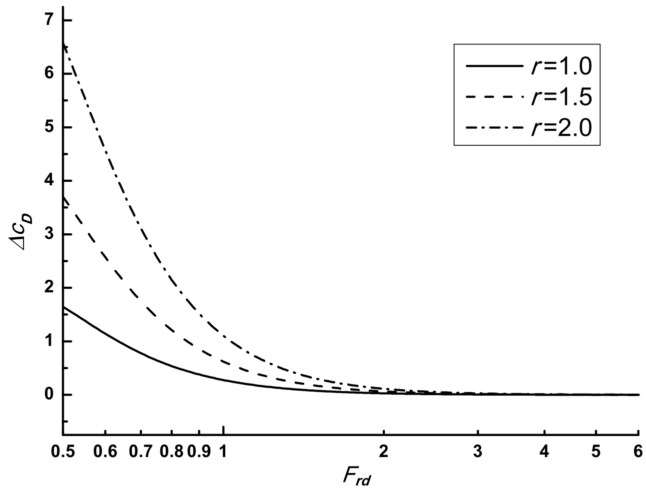

Fig. 2. A theoretical prediction of c_D of the Rankine ovoids as different length-to-radius ratio r.

of the Rankine ovoids with the different length-to-radius ratios.

Fig. 1 shows that the theoretical and experimental results are in good agreement. Based on these results the constant C in (27) can be determined as 1.104 experimentally. Fig. 2 shows that as the internal Froude numbers are larger than 2, there exist an asymptotic state, i.e. all the curves become almost a horizontal straight line, whereas when the internal Froude number is smaller than 2, the drag coefficients disperse by the length-to-radius ratios. Generally, the larger the length-to-radius ratio is, the larger the drag coefficient is. At the same time, the differences of the constants C between the sphere and the Rankine ovoids are very small, hence this fact led us to draw a conclusion, that is no matter a sphere and a Rankine ovoid, (30) holds if F_{R_0} or F_{χ_0} is larger than 2. However, if they are smaller than 2, (30) does not hold anymore.

Finally, it should be pointed out that even though the paradox of infinite wave drag is removed by the regulation, introduction of some non-local sources is necessary in further studies.

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